جامعة بيرزيت

Birzeit University

Computer Science Department – Engineering and Information Technology Faculty
Comp233, Discrete Mathematics, Dec 5, 2019
Winter 2019

Student Name: Key

Student ID: ____________

➢ **Instructor: Mr. Murad Njoum**

Question 1 (26%) :

1) *The inverse of function* $f(x) = x^3 + 2$ *is* ____________
   a) $f^{-1}(y) = (y - 2)^{1/2}$
   b) $f^{-1}(y) = (y - 2)^{1/3}$
   c) $f^{-1}(y) = (y - 1)^{1/3}$
   d) $f^{-1}(y) = (y - 1)$

2) *The function* $f(x) = x^3$ *is bijection from R to R. Is it True or False?*
   a) True
   b) False

3) *Which of the following function* f: $Z \times Z \rightarrow Z$ *is not onto?*
   a) $f(a, b) = a + b$
   b) $f(a, b) = a$
   c) $f(a, b) = |b|$
   d) $f(a, b) = a - b$

4) *A function is said to be* ______________ *if and only if* $f(a) = f(b)$ *implies that* $a = b$ *for all a and b in the domain of f.*
   a) $One - to - many$
   b) $One - to - one$
   c) $Many - to - many$
   d) $Many - to - one$

5) *4. If the number of binary subsets of a set are 4 then the number of elements in that sets are*
   a) 1
   b) 2
   c) 3
   4) 4

6) *Let the set be* $A = \{a, b, c, \{a,b\}\}$ *then which of the following is false*
   a) $\{a, b\} \in A$
   b) $a \in A$
   c) $\{a\} \in A$
   d) $b, c \in A$

7) *The set containing all the collection of subsets is known as*
   a) *Subset*
   b) *Power set*
   c) *Union set*
   d) *None of the mentioned*

8) *If set A and B have 3 and 4 elements respectively then the number of subsets of set (A X B) is*
   a) 1024
   b) 2048
   c) 512
   d) 4096

9) *If $A \subseteq B$ then $A \times C \subseteq B \times C$ the given statement is*
   a) *True*
   b) *False*

10) *Let A = {1, 2, 3} and B = {x, y, z}. Consider the relations R = {(1,x),(2,x)} and S = {(1,x),(1,y),(2,z),(3,y)}. The S is*

a) *One − to − One*
b) *Onto*
c) *Correspondence*
d) *Not function*

Bonus *(I am not obliged to answer the question)*

11) *Evaluate the performance of comp233 instructor at this semester?*
a) 90 − 100% b) 80 − 89 % c)70 − 79% d) 60 − 69% e) Under 60%

12) *What grade you expect at this course?*
a) 90 − 99% b) 80 − 89 % c)70 − 79% d) 60 − 69% e) Under 60%

13) *How do you evaluate yourself in class attendance?*
a)Always b) Noramal c)rearly d) never

Question 2 (20%):

I) Suppose A = {1, 2} and B = {2, 3}. Find each of ($\mathcal{P}$: *is power set*) the following:

(2%) a) $\mathcal{P}(A)$

(3%) b) $\mathcal{P}(A \cap B)$

(5%) c) $\mathcal{P}(A \cup B)$

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2) *The function $f(x) = x^2$ is bijection from R to R. Is it True or False?*
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3) *Which of the following function f: $Z \times Z \to Z$ is not onto?*
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   c) $f(a,b) = b$
   d) $f(a,b) = |a - b|$

4) *A function is said to be* ____________ *if and only if $f(a) = f(b)$ implies that $a = b$ for all a and b in the domain of f.*
   a) $One - to - many$
   b) $Many - to - one$
   c) $Many - to - many$
   d) $one - to - one$

5) *4. If the number of binary subsets of a set are 8 then the number of elements in that sets are*
   a) 1
   b) 2
   c) 3
   4) 4

6) *Let the set be A= {a , b, c, {a,b}} then which of the following is false*
   a) $\{a, b\} \in A$
   b) $\{b\} \in A$
   c) $a \in A$
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7) *The set containing all the collection of subsets is known as*
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8) *If set A and B have 2 and 5 elements respectively then the number of subsets of set (A X B) is*
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   a) *True*
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10) *Let A = {1, 2, 3} and B = {x, y, z}. Consider the relations R = {(1,x),(2,x)} and S = {(1,x),(1,y),(2,z),(3,y)}. The S is*

a) *Sirjective*
b) *Bijective*
c) *Correspondence*
d) *Not function*

Bonus *(I am not obliged to answer the question)*

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Question 2 (20%):

I) Suppose A = {1, 2} and B = {2, 3}. Find each of ($\mathcal{P}$: *is power set*) the following:

(2%) a) $\mathcal{P}(A) = \{\phi, \{1\}, \{2\}, \{1,2\}\}$

(3%) b) $\mathcal{P}(A \cap B) = P(\{2\}) = \{\phi, \{2\}\}$

(5%) c) $\mathcal{P}(A \cup B) = P(\{1,2,3\}) = \{\phi, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}, \{1\}, \{2\}, \{3\}\}$

II) Let $S_i = \{x \in R \mid 1 < x < 1 + \frac{1}{i}\} = (1, 1 + \frac{1}{i})$, *for each positive integer find:*
(3 %,3%,4%)

a. $\bigcup_{i=1}^{4} S_i = ?$ $S_1 \cup S_2 \cup S_3 \cup S_4 = (1,2) \cup (1,3/2) \cup (1,4/3) \cup (1,5/4) = (1,2)$

b. $\bigcap_{i=1}^{4} S_i = ?$ $S_1 \cap S_2 \cap S_3 \cap S_4 = (1,2) \cap (1,3/2) \cap (1,4/3) \cap (1,5/4)$

$= (1, 5/4)$

c. $\bigcup_{i=1}^{\infty} S_i = ?$ $S_1 \cup S_2 \cup S_3 \cup S_4 \cdots \cup S_n = (1,2) \cup (1,3/2) \cup (1,4/3)$

$\cup (1,5/4) \cdots \cup (1, 1+1/n)$

$\bigcup_{i=1}^{\infty} S_i = \lim_{n\to\infty} \bigcup^{n} S_i = \lim_{n\to\infty} \bigcup (1,2) = (1,2)$

Question 3 (30%, 15% each):

Define $H : \boldsymbol{R} \times \boldsymbol{R} \to \boldsymbol{R} \times \boldsymbol{R}$ *as follows:*
$H(x,y) = (x+1, 2-y)$ *for every* $(x,y) \in \boldsymbol{R} \times \boldsymbol{R}$.
a. *Is H one-to-one? Prove or give a counterexample.*

Yes, H is One-to-One function since.

Let $(x_1, y_1), (x_2, y_2) \in \mathbb{R} \times \mathbb{R}$ such that

if $H(x_1, y_1) = H(x_2, y_2)$

we want to show that $x_1 = x_2$, $y_1 = y_2$

Hence, $H(x_1, y_1) = H(x_2, y_2)$

$\Rightarrow (x+1, 2-y_1) = (x_2+1, 2-y_2)$

$\therefore x_1 + 1 = x_2 + 1 \Rightarrow \therefore \boxed{x_1 = x_2}$

& $2 - y_1 = 2 - y_2 \Rightarrow \boxed{y_1 = y_2}$

So, H function is One-to-One ✗

b. *Is* H *onto? Prove or give a counterexample.*

Yes, H is Onto function

Let $(x, y) \in \mathbb{R} \times \mathbb{R}$. We need $(r, s) \in \mathbb{R} \times \mathbb{R}$ such that $H(r, s) = (x, y)$. Hence from definition, let us solve $H(r,s) = (x,y)$

$\Leftrightarrow (r+1, 2-s) = (x, y) \Rightarrow r+1 = x,\ 2-s = y$

$\therefore r = x-1,\ s = 2-y$, So $(r,s) = (x-1, 2-y) \in R \times R$

Since; if we substitute $(r,s) = (x-1, 2-y)$ in H function we get $H(r,s) = (x,y)$ Hence H is Onto #

Question 4 (30%, 15% each):

a) *Construct an algebraic proof for the given statement.*

For all sets A, B, and C,
$(A \cup B) - (C - A) = A \cup (B - C)$. **Cite a property from every step?**

$$
\begin{aligned}
(A \cup B) - (C - A) &= (A \cup B) \cap (C - A)^c && \text{by difference law} \\
&= (A \cup B) \cap (C \cap A^c)^c && \text{by difference law} \\
&= (A \cup B) \cap (A^c \cap C)^c && \text{by commutative law} \\
&= (A \cup B) \cap ((A^c)^c \cup C^c) && \text{by DeMorgan's law} \\
&= (A \cup B) \cap (A \cup C^c) && \text{by distributive law} \\
&= A \cup (B \cap C^c) \\
&= A \cup (B - C) && \text{by difference law}
\end{aligned}
$$

#

b) *Use an* ***element argument*** *to prove this statement Assume that all sets are subsets of a universal set U.* **Justify you each step**

For all sets A, B, and C
$(A - B) \cup (C - B) = (A \cup C) - B$

1) Let x (pbac). Such that $x \in (A-B) \cup (C-B)$
we want to show that $x \in ((A \cup C) - B)$ is true.
by definition of subset
$x \in (A-B) \cup (C-B)$ and by diffenition of union. $x \in A-B$ or $x \in C-B$

For: $x \in A - B$ (case 1) by diffenition of difference
$x \in A$ and $x \notin B$
By the difinition of union and using $x \in A$: $x \in A \cup C$
By difinition of differece, using $x \in A \cup C$ and $x \notin B$
so, $\boxed{x \in (A \cup C) - B}$

Second case ($x \in C - B$): by difeintion of difference $x \in C$ and $x \notin B$. by difinition of union and using $x \in C$, so $x \in A \cup C$. by diffition of differece using $x \in A \cup C$ and $x \notin B$ so, $\boxed{x \in (A \cup C) - B}$

2) Let $x \in (A \cup C) - B$, we want to show $x \in (A-B) \cup (C-B)$ is true.
$x \in (A \cup C) - B \Rightarrow$ by def. of dif $\Rightarrow x \in A \cup C$ and $x \notin B$
by dif of union, ($x \in A$, or $x \in C$) and $x \notin B$.
First case: $x \in A$, and $x \notin B$. by def of dif : $x \in A - B$
by def of union $x \in A - B$, so $\boxed{x \in (A-B) \cup (C-B)}$
Second case: $x \in C$ and $x \notin B$. by def of dif : $x \in C - B$
by def of union $x \in C - B$, $\boxed{x \in (A-B) \cup C - B}$
$\boxed{\rightarrow x \in (A-B) \cup (C-B)}$ Both Side is the so #